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Impact of ramp-up on the optimal capacity-related reconfiguration policy

A. Matta[1], M. Tomasella[1], A. Valente[1]

[1] Dipartimento di Meccanica – Sezione Tecnologie Meccaniche e Produzione, Politecnico di Milano, Piazza Leonardo da Vinci 32, 20133, Milano, Italy.

Corresponding author e-mail contact: maurizio.tomasella@polimi.it

Additional e-mail contacts: andrea.matta@polimi.it, anna.valente@polimi.it

Abstract: This paper presents an optimal solution, based on Markov Decision Theory, for the problem of optimal capacity-related reconfiguration of manufacturing systems, under stochastic market demand. Both capacity expansion and reduction are considered. The solution quantitatively takes into account the effect of the ramp-up phenomenon, following each reconfiguration, on the optimal policy. A closed-form solution is presented in the case product demand is independently and generally distributed over time. A real case concerning a flexible manufacturing line in the automotive sector is shown, to prove that ignoring the ramp-up effect in the decision process can lead to significant increases in the overall costs.

Keywords: Reconfigurable Manufacturing Systems, Ramp-up, Capacity Planning, Markov Decision Problem, Dynamic Programming

1 Introduction

Ramp-up is defined in the literature as “the time interval it takes a newly introduced or just reconfigured production system to reach sustainable, long-term levels of production, in terms of throughput and part quality, considering the impact of equipment and labor on productivity” (Koren, *et al.*, 1999). One of the alternative definitions presents production ramp-up as “the period during which a manufacturing process makes the transition from zero to full-scale production at targeted levels of cost and quality” (Clark and Fujimoto 1991).

The ability to successfully ramp-up the production when new products are to be introduced has become a critical issue for many manufacturing companies, especially for Original Equipment Manufacturers and their suppliers. Moreover, product life cycles are shortening and individualised customization of products is increasing, thus leading to more frequent production ramp-ups than before. Hence, this frequency pushes manufacturers to manage production ramp-ups both in a time and cost-efficient manner. To achieve a fast pay-back of investments in new product designs and their related production facilities, companies must reduce not only their product development time (time-to-market, Figure 1) but also the lead-time to achieve satisfactory manufacturing volumes, costs, and quality (time-to-volume, Figure 1). The basic difference between time-to-market and time-to-volume is that the former ends with the beginning of commercial production whereas the latter explicitly includes the period of production ramp-up.

Whereas many works have investigated time-to-market, the topic of time-to-volume has received less attention. Yet the timing of revenues mainly depends on time-to-volume, while development expenses are generally concentrated before product launch. This gives time-to-volume high leverage in determining the net present value related to the lifecycle of production systems (Terwiesch and Bohn, 1999). In high-tech short-lifecycle industries, for instance semiconductors or hard-disk drives industries, the portion of lifecycle that the product spends in ramp-up conditions can be very large and must be carefully considered. Since in these industries prices generally fall very rapidly, to achieve high volumes in the earliest phases becomes crucial for getting high financial payoffs (Hatch and Macher, 2004).

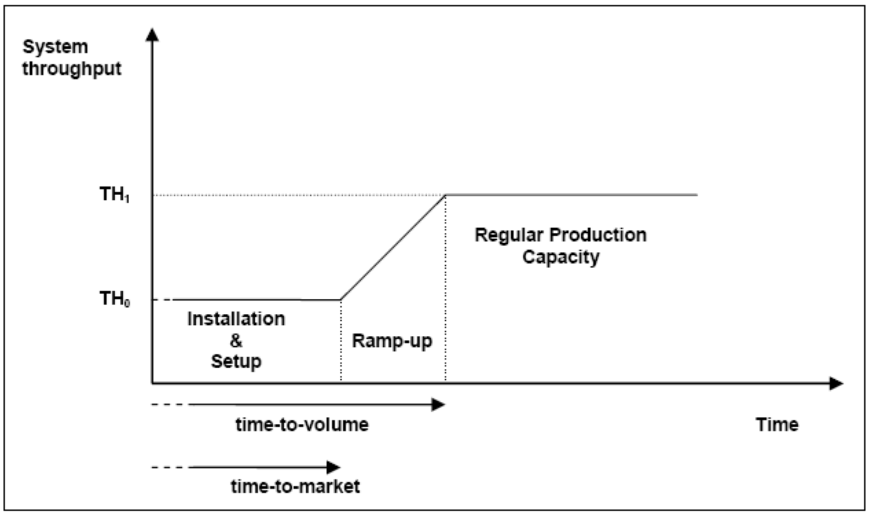

Figure 1: Time-to-market, time-to-volume and the ramp-up phenomenon

The management of the production system ramp-up phenomenon is mainly related to three issues. The first one concerns the analysis and identification of the significant factors affecting the duration of the ramp-up phase and its related costs. The second issue regards methods and tools for guiding the production manager in the process of reducing the duration of the ramp-up phase, by enhancing as quickly as possible the quality of production output. The last one deals with methods and tools for aiding the system designer in assessing the system ramp-up during the system configuration/reconfiguration. The knowledge of how ramp-up affects the effectiveness of the reconfiguration policy implemented by an enterprise would help system designers to make the right decisions and choose the best system configuration/reconfiguration policies.

Most of the approaches to ramp-up analysis existing in the literature are devoted to the first two issues, while only a few deal with the latter. In the following, some of the works in these different areas will be briefly cited.

In the area related to the analysis of the most important factors affecting system ramp-up, the majority of works concerns the analysis of specific industrial cases and the collection of the related research experiences. In (Mileham, *et al*, 2004) the authors start from the analysis of production contexts belonging to a variety of industries. This work seeks for methods for predicting changeover duration at any given product change, and for indicating the influence of individual parameters on changeover time. To cope with these objectives the authors use two approaches: Decision Tree Induction and Artificial Neural Networks. Results concerning the measure of the relative importance of parameters are deduced by perturbing the entire training data one parameter at a time and then averaging the change in output due to these perturbations. The product type can be seen to be the most significant parameter, with team and shift also influential. It was noted that omission or inclusion of these parameters as training data had a large influence on model accuracy.

Li and Rajagopalan in (Li and Rajagopalan, 1998) assert that the most influencing factors are related to learning processes, since learning influences the system set-up times and other phenomena occurring during the ramp-up phase. In (Li and Rajagopalan, 1998) empirical studies are provided to support the consideration that learning represents the link between quality improvement and productivity increase. The same assertion is also motivated in (Hatch and Macher, 2004), which presents a model of knowledge creation and deployment to solve the trade-off between technological innovation and manufacturing performance. The authors highlight that this trade-off is mitigated by knowledge management, since higher levels of cumulative production, skills of human resources and organizational practices are necessary to accelerate learning and to reduce the product time-to-market.

Among the works in the area related to the ramp-up reduction, Terwiesch and Bohn start analyzing factors such as machine breakdowns and slow set-ups, which reduce production rates and yields, and relate these factors to the learning process (Terwiesch and Bohn, 2001). Afterward, an empirical relationship between capacity utilization and yields has been provided, allowing to improve yields during the ramp-up phase and to reduce overall ramp-up time.

Moving to the last area, although the importance of system ramp-up is deeply perceived by companies, it seldom happens that the production system designer quantitatively considers the phenomenon in designing new systems or in redesigning existing ones. Difficulties in precisely evaluating ramp-up time and cost depend on the heterogeneous factors which are involved: the technology implemented in the system and its machines/material handling devices (production machining, forming, joining, etc.), the system architecture (e.g. dedicated systems, flexible systems, etc.), the company's manufacturing strategy and its production planning and management policies and daily practice. This basically means that ramp-up duration and cost vary in accordance to the specific production context.

Among the few works in the area of assessing the impact of the ramp-up phenomenon on the system reconfiguration phase, (Deif and ElMaraghy, 2006) and (Deif and ElMaraghy, 2007) describe the effect of reconfiguration costs, which also include ramp-up costs, on capacity scalability planning horizon and overall costs in RMSs (Reconfigurable Manufacturing Systems). A genetic algorithm (GA) technique for generating an optimal capacity scalability schedule was developed; the level of the capacity to be scaled and the cost of the capacity scalability schedule in RMSs are related to the cost of system reconfiguration. Thus, the cost-effective implementation of an RMS highly depends on decreasing the cost of reconfiguration of these systems.

Finally, (Amico, *et al.*, 2006) proposes a capacity planning model for RMSs in which investment decisions are modeled through the joint use of the Discounted Cash Flow (DCF) and the Real Option Analysis (ROA) techniques. The authors model the "operating flexibility", a kind of flexibility giving project managers options to revise decisions in response to production changes, taking into consideration the system ramp-up phenomenon. The application of both DCF and ROA allows to determine an optimal Expected Net Present Value (ENPV) solution considering ramp-up costs, plus the value of all the options embedded in the project. The developed tool has been utilized to compare the value of an expansion option for a Dedicated Manufacturing Line, a Flexible Manufacturing System and a Reconfigurable Manufacturing System making a single product. The result coming from the ENPV application highlights the advantage of choosing an RMS. The authors claim that thanks to RMS scalability and convertibility, the real option of increasing the capacity can have a significant value, up to 6% of the NPV.

The analysis of the issues related to the system ramp-up phase supports the following idea: neglecting the existence of ramp-up in the reconfiguration problem can lead to underestimate investment and operating costs. The research proposed within this paper mainly deals with the ramp-up modeling and the assessment of its impact on the reconfiguration policies. The key idea behind the work is that, by considering the weight of the ramp-up phase more profitable decisions can be made. In particular, the ramp-up phenomenon has been considered within the optimal policy to decide when, how and how much to reconfigure the production system capacity, under uncertain market demand (Luss, 1982; Rocklin and Kashper 1984; Asl and Ulsoy, 2002a; Asl and Ulsoy 2002b).

The present work represents an extension of (Matta *et al*, 2007a) in which a closed-form solution for the optimal capacity-related reconfiguration problem has been provided: therein, the volume demand was assumed independently and uniformly distributed over time. In this work the same authors will present the complete model, for which the optimal policy can be calculated for any kind of continuous univariate distribution. The considered problem is relevant in all those contexts where the demand is affected by a high degree of uncertainty, in particular where the ramp-up effect is *sensibly not negligible*. A more precise quantification of this sensible non-negligibility will be also provided.

The outline of the paper looks as follows. Section 2 provides a thorough formulation of the problem. Section 3 derives the optimal policy and analyzes the main analytical results implied by the use of such a policy. Section 4 presents some numerical considerations on a real case concerning a flexible manufacturing line in the automotive sector, to further discuss the potential related to the use of the presented policy. Finally, conclusions are drawn in Section 5.

2 Formulation of the Reconfiguration Problem

This paper aims at considering the impact of the ramp-up phenomenon on the system reconfiguration. A given production system configuration can be defined as the set of resources and logics (machine tools, buffers, production policies, etc.) required to satisfy a given production problem. In this work, only the reconfiguration of system capacity is taken into consideration, where system capacity is defined as the amount of good finished pieces produced by the system in each period of a given length.

After having defined the production requirements evolution, in terms of stochastic evolution of the market demand, the decision maker has to identify in each demand scenario the optimal capacity level to be achieved. In particular, the amount of capacity modification, either expansion or reduction, represents the decision variable of the optimization problem. Based on the decision made, a related cost is incurred depending also on the realized scenario. A multi-period approach is considered: given the current level of system capacity, the level of capacity expansion/reduction leading to the optimal value of a given multi-period cost function is chosen.

2.1 Basic assumptions and notation

This section provides a detailed description of the basic set of assumptions and related notations of the model.

First of all, only one product type is considered and the product evolution is described only in terms of product demand evolution, as derived by market forecasts. Therefore, modifications of one or more of the product features and other technological aspects are not considered in the present paper. This aspect becomes indeed crucial for the problem of the optimal functionality-related reconfiguration of the system (together with the capacity-related one), where functionality is defined as the vector of system characteristics enabling the production process (Matta, *et al.*, 2007b, Koren, *et al.*, 1999).

Moreover, only physical (hard, as defined in ElMaraghy 2005) types of capacity-related reconfiguration are considered. This means that only reconfigurations at hardware level are taken into account by the model variables and parameters, though, in principle, logical (soft) reconfigurations, involving only software and management aspects of the production system, could also be considered. As a consequence, the present model can be used to decide when/how to change the number of machines in a system, such as the number of machining centers in a Flexible Manufacturing System, but not to decide for instance about the possible use of additional production shifts or subcontracting.

The planning horizon is assumed finite, with $k \in \{0,1,\ldots,N\}$ being the related discrete time index. The length of each period k is fixed and equal to T_k. The market demand $\{D_k\}_{k=0,1,2,\ldots,N-1}$ in period k is assumed to be stochastic, with a generic, continuous and non-negative distribution $\{\Psi_k\}_{k=0,1,2,\ldots,N-1}$. Demand distributions are considered to be defined over intervals of the form $[\delta_k, \Delta_k]$. Both δ_k and Δ_k have positive finite values. No constraint exists on the shape or nature of the demand distribution. Moreover, demand distributions in different periods are assumed to be independent from one another, i.e. Ψ_k and Ψ_{k+s} are independent on each other, for any value of k and s.

System capacity at time k, denoted with C_k, takes values in $\mathbb{R}^+ \cup \{0\}$. $\{X_k\}_{k=0,1,2,\ldots,N-1}$ represents the decision variable, i.e. the amount of capacity to be added/subtracted, according to the decision taken at time k. $C_{k+1} = C_k + X_k$ is the state transition equation, leading the system from the "state" C_k at time k to the "state" C_{k+1} at time $k+1$. The phases of installation and set-up of the added capacity, or of removal of the subtracted capacity, end by the start of the subsequent period. To be noted that the link between system capacity as an overall number, the production mix to be produced and the production plan (with the specific operations) is not considered in the present paper. The model presented here works in fact on a lower level of detail. It is important however to underline that, after having properly planned the needed production capacity, the production manager will have to move into the detailed design of the production system. One should thus be able to assess and analyze the actual capacity of a given system configuration, given the information on the production plan and the needed operations, to check if the considered configuration meets (or not) the previously planned capacity. A new interesting mathematical approach for this purpose is presented by Koltai and Stecke (Koltai and Stecke 2007).

The following parameters are also known to the decision maker:

- *unit production cost* (γ_P), i.e. the full manufacturing cost of each product;

- *price* (P), i.e. the reward related to selling a product unit;
- *unit penalty cost* (γ_S), i.e. the unit cost associated to each product requested by the market but not produced by the system (no inventory management is considered in the model). It can be thought of as a penalty cost defined by contract with the customer or, in case outsorcing can fill the demand gap on request, as the unit negative profit.
- *unit holding cost* (γ_H), i.e. the overhead cost per unit of capacity in each time period. It is composed of the costs for maintaining and staffing each unit of capacity available in each period.

Finally, for simplicity, when a system reconfiguration occurs (i.e. $X_k \neq 0$), the profile of the system throughput (measured in pieces/day), can be represented by the function illustrated in Figure 2. In this case, when a reconfiguration has occurred, the steady-state throughput (TH_k^{SS}) related to the new configuration is reached only after a certain time, namely τ_k. Moreover, it is assumed that $\tau_k < T_k$, i.e. that the duration of the ramp-up period never exceeds the duration of the whole period. If, for period $k+1$, no modification of system capacity is foreseen (i.e. $X_k = 0$), no ramp-up penomenon takes place, and the system is assumed to be immediately capable of providing throughput $TH_{k+1}^{SS} = TH_k^{SS}$.

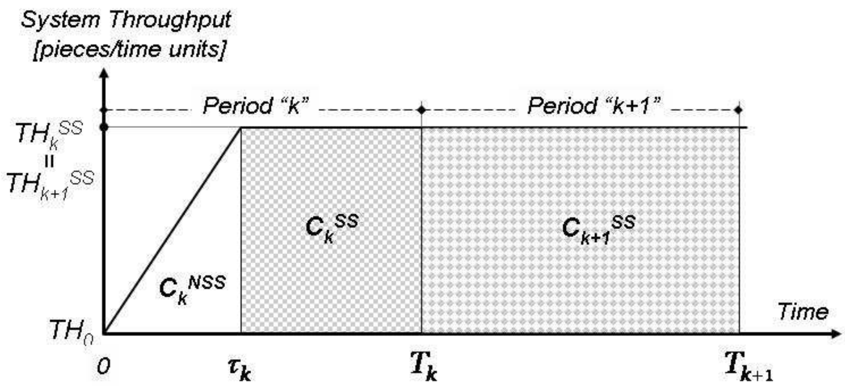

Figure 2: Assumptions on the profile of the system throughput during the ramp-up phase

Each period can thus be generally divided into two distinct sub-periods.

The first sub-period ($t \leq \tau_k$) represents the system ramp-up phase, whose main feature is the non-steady-state production. This last is a consequence of the system inability to reach its full production potential, –because of higher frequency of machine/system breakdowns, higher scraps, reworks, and poor understanding of the best way to conduct the system in its new configuration. It is assumed that all these inefficiencies during ramp-up can be expressed by a higher unit production cost; for simplicity this cost is equal to $\gamma_m \geq \gamma_P$ for each product unit produced in this phase. The actual value of the unit production cost during ramp-up will in general show a decreasing profile and will be higher than the steady-state unit production cost for any $t \leq \tau_k$. To keep things simple, though it would not be essential, the hypothesis of a constant (with respect to time) γ_m will hold in the following pages.

The second sub-period ($t \geq \tau_k$) represents the steady-state of the system. Each product is manufactured incurring a unit production cost equal to γ_P. The production rate is assumed to be constant and equal to TH_k^{SS}.

In each period k, the actual value of the system capacity, denoted with C_k^R (the ^{R} in the apex stands for "real"), will therefore be in general lower than the theoretical value C_k (in general, C_k^R will be an ε_k fraction of C_k).

In fact, C_k^R is composed (Figure 2) of C_k^{SS}, i.e. the total actual system capacity during the steady-state sub-period, and of C_k^{NSS}, which represents the actual system capacity during the non-steady-state sub-period (i.e. during ramp-up).

In particular, $C_k = TH_k^{SS} \cdot T_k$ and, for the considered throughput profile, $\varepsilon_k = C_k^R / C_k = 1-(\tau_k/(2T_k))$. Please note that $(1-\varepsilon_k)$ is the percentage of capacity loss related to the ram-up effect. Concluding, τ_k is assumed to range in the interval $(0,T_k)$ and ε_k takes values in $(1/2,1)$: indeed, when $\tau_k \rightarrow 0$, $\varepsilon_k \rightarrow 1$, while for $\tau_k \rightarrow T_k$, $\varepsilon_k \rightarrow 1/2$.

2.3 Cost model description

The manufacturer has to decide, at each period k, which capacity the production system must have in order to face the market uncertainties. If the system capacity is expanded or reduced, the system is said to be reconfigured. The sequence of reconfiguration decisions is called the *reconfiguration policy*, chosen by pursuing the minimization of a given total expected discounted cost function over the whole planning period. In the following, the structure of this cost function is described in detail.

The total expected discounted cost represents the accumulation over time of the *control cost* (CC_k), which models the cost incurred in each period k as a consequence of the decision (X_k) made in the same period, and also depending on the value of C_k. CC_k is defined as the sum of the *expected operating cost* (EOC_k) incurred in the period, due to the fact that the system operates with system capacity C_k, and of the *capacity management cost* (M_k), directly related to the decision made in period k:

$$CC_k(C_k, X_k) = EOC_k(C_k) + M_k(X_k) \tag{1}$$

The expected operating cost in period k is the sum of the *expected production cost* (EPC_k), of the *expected ramp-up production cost* ($ERUPC_k$), of the *expected shortage cost* (ESC_k), and of the *expected holding cost* (EHC_k). All these expected values are computed with respect to the possible values of D_k in period k, given the demand distribution $\Psi_k(D_k)$. Formally:

$$EOC_k(C_k) = EPC_k(C_k) + ERUPC_k(C_k) + ESC_k(C_k) + EHC_k(C_k) \tag{2}$$

$$EPC_k(C_k) = E_{D_k}\left[\left(\gamma_P - P\right) y_k\right] \tag{3}$$

$$ERUPC_k(C_k) = E_{D_k}\left[\left(\gamma_m - \gamma_P\right) w_k\right] \tag{4}$$

$$ESC_k(C_k) = E_{D_k}\left[\gamma_S z_k\right] \tag{5}$$

$$EHC_k(C_k) = E_{D_k}\left[\gamma_H C_k\right] = \gamma_H C_k \tag{6}$$

Quantity y_k represents the production level in period k. Since no inventory management is allowed, or equivalently that all produced pieces are assumed to be in any case sold in the market, this quantity is defined as:

$$y_k = \min\left(D_k, C_k^{R}\right) \tag{7}$$

Quantity w_k, representing the total amount of pieces produced during ramp-up, is defined as:

$$w_k = \min\left(D_k, C_k^{NSS}\right) \tag{8}$$

Quantity z_k is equal to the level of unmet demand in period k, according to:

$$z_k = \max\left(0, D_k - C_k^{R}\right) \tag{9}$$

The capacity management cost is defined as:

$$M_k(X_k) = \overbrace{L(X_k)\cdot(E+e\cdot X_k)}^{\text{expansion cost}} + \overbrace{L(-X_k)\cdot(R+r\cdot X_k)}^{\text{reduction cost}} \tag{10}$$

where:

$$L(X_k) = \begin{cases} 1 & X_k > 0 \\ 0 & X_k \le 0 \end{cases} \tag{11}$$

E (e) and R (r) represent the fixed (variable) expansion and reduction costs respectively. The term E models the setup cost to install additional system capacity, and e is the corresponding unit ordering cost. On the other hand, the term R models the one-period labor cost to uninstall excess system capacity, and r is the reward for selling one unit of system capacity. Other costs related to system reconfiguration and not explicitly mentioned or defined in the previous pages, can be considered as a portion of the constant terms E or R. Herein it is assumed $e \geq r$: this assumption is considered reasonable for many cases of practical interest.

Table 1. Definition of the quantities involved in the computation of operating costs

$\mathbf{C^{NSS}_k}$	$\mathbf{C^R_k}$	$\mathbf{D_k}$	$\mathbf{w_k}$	$\mathbf{y_k}$	$\mathbf{z_k}$	$\boldsymbol{D_i}$
$C^{NSS}_k \geq \Delta_k$	$C^R_k \geq \Delta_k$	$\forall D_k$	D_k	D_k	0	D_6
$\delta_k \leq C^{NSS}_k \leq \Delta_k$	$C^R_k \geq \Delta_k$	$\delta_k \leq D_k \leq C^{NSS}_k$	D_k	D_k	0	D_5
		$C^{NSS}_k \leq D_k \leq \Delta_k$	C^{NSS}_k	D_k	0	
	$\delta_k \leq C^R_k \leq \Delta_k$	$\delta_k \leq D_k \leq C^{NSS}_k$	D_k	D_k	0	D_4
		$C^{NSS}_k \leq D_k \leq C^R_k$	C^{NSS}_k	D_k	0	
		$C^R_k \leq D_k \leq \Delta_k$	C^{NSS}_k	C^R_k	$D_k - C^R_k$	
$C^{NSS}_k \leq \delta_k$	$C^R_k \geq \Delta_k$	$\forall D_k$	C^{NSS}_k	D_k	0	D_3
	$\delta_k \leq C^R_k \leq \Delta_k$	$\delta_k \leq D_k \leq C^R_k$	C^{NSS}_k	D_k	0	D_2
		$C^R_k \leq D_k \leq \Delta_k$	C^{NSS}_k	C^R_k	$D_k - C^R_k$	
	$C^R_k \leq \delta_k$	$\forall D_k$	C^{NSS}_k	C^R_k	$D_k - C^R_k$	D_1

At a generic period k, given the values for C_k, τ_k (or equivalently ε_k), δ_k and Δ_k, quantities y_k , w_k and z_k take different values (Table 1). Six different situations exist, depending on the 4-tuple (C_k, τ_k, δ_k, Δ_k). For each of these situations, a specific form of each of the operating cost components is defined (denoted in the table with D_1, …, D_6).

Again considering period k, one out of two mutually exclusive cases occurs, depending on whether $\delta_k/\Delta_k \leq (1-\varepsilon_k)/\varepsilon_k$ or its contrary holds (Figure 3). This has an impact on the analytical form of the operating costs functions to be considered in the derivation of the optimal solution, but it does not jeopardize the main characteristics of the same functions, such as their continuity over their supports and above all the existence of the optimal solution.

It can be easily demonstrated that EPC_k, $ERUPC_k$, ESC_k and EHC_k (and thus EOC_k), are all continuous functions of C_k, for every $C_k \in [0,+\infty)$. More precisely, they are continuous functions of C_k, over any compact set of the form $[0, M_k]$, with a generic $M_k < +\infty$. This, by the well known Weierstrass theorem, yields the existence of a minimum for all of these functions, in any compact set of the form $[0, M_k]$, with $M_k < +\infty$. The present characterization, in particular for EOC_k, is fundamental for the existence of the optimal solution to the capacity-related reconfiguration problem.

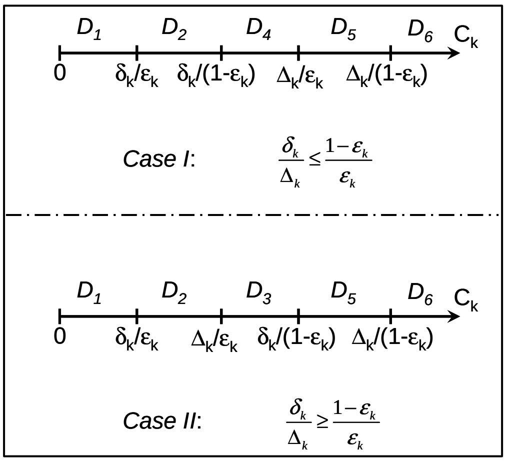

Figure 3: Definition of cases *I* and *II*

Figure 4 shows the profiles of the operating cost functions related to the case study which will be presented in Section 4 (data in Table 2).

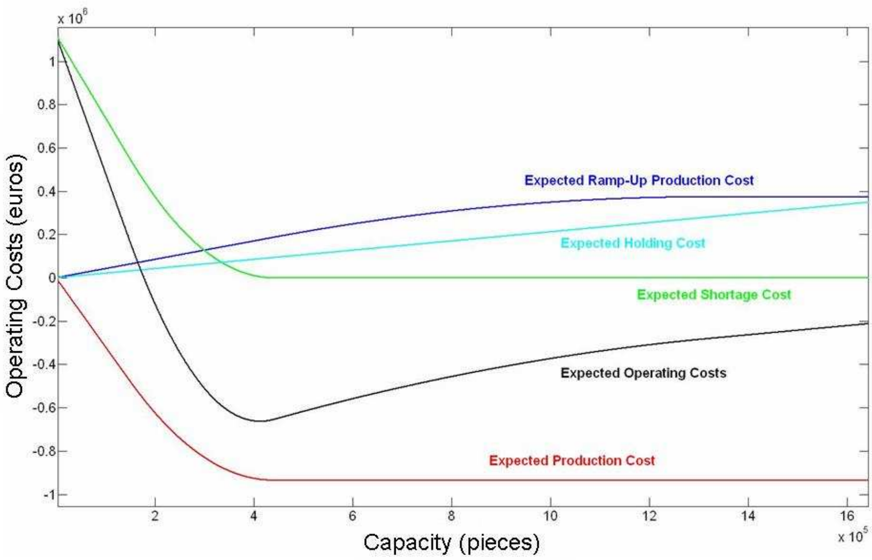

Figure 4: Example of profiles of the operating costs functions (*Case I*)

2.4 Definition of the optimal capacity-related reconfiguration problem

The described problem is a well-behaved Markov Decision Problem (MDP) over a discrete-time finite horizon of N periods (Kumar 1986) (Cassandras and Lafortune 2001), where at each period k the manufacturer has to decide the *control action* X_k, with $k=0,\ldots, N-1$. The values of the system capacity and of the product demand requested by the market jointly define the *state* of the *process* at each period. Since product demand was assumed to take values in a continuous domain, the number of possible states at each period is infinite. Whenever the process enters a new period, it is assumed that the manufacturer observes the new state, incurs the operating costs related to that state and takes the control action X_k.

Given an initial state (C_0 ,$\Psi_0(D_0)$), the problem consists in finding the optimal decision sequence, i.e. the *optimal reconfiguration policy*, denoted by $\pi^* = \{X_0^*, X_1^*, X_2^*,\ldots, X_{N-1}^*\}$, which minimizes the following total expected discounted cost function:

$$V^*_\pi(C_0, \Psi_0(D_0)) = \min_\pi \left\{ \sum_{k=0}^{N-1} \alpha^k \left[EOC_k(C_k) + M_k(X_k) \right] - \alpha^N \gamma_N C_N \right\} \quad (12)$$

where $C_N \geq 0$ is the capacity available at the beginning of period N, $0 \leq \alpha \leq 1$ is a discount factor and $\gamma_N \geq 0$ represents the salvage value at which each of the units of the terminal capacity can be sold (at least an estimation of γ_N is assumed to be available since the starting period). Moreover, it is assumed that the total value of system capacity at period N is a convex function of capacity C_N; this last assumption is strictly related to the existence of the optimal reconfiguration policy.

3 Optimal capacity expansion/reduction levels

A fundamental solution technique for MDPs is based on Dynamic Programming Theory (Bellman, 1957). By using this technique, it is generally possible to derive a policy which is simultaneously optimal for every initial state. Moreover, in situations where decisions are made in stages while gathering information on the state of the process, a so-called *closed-loop* minimization of the total expected discounted cost takes place (Bertsekas, 1987). This means that, when updated information on the state of the process is available, this information can be exploited to make better decisions. In cases such as the MDP defined above, the decision maker is interested not in finding a sequence of numerical values for the optimal control action X_k*, but rather in finding a sequence of functions $X_k*(C_k, \Psi_k)$, mapping the process state into the optimal control action, for every state and for every period *k*. DP can be effectively used for this purpose, and Sub-section 3.1 is devoted to deriving such a kind of optimal policy, while Sub-section 3.2 provides a first insight into the capabilities of the model and into the meaning of the optimal policy.

3.1 Derivation of the optimal policy

In the following, the closed-form solution of the optimal capacity-related reconfiguration problem is derived. The procedure strongly relies on DP theory, assumptions and notation. For reasons of space, the derivation cannot be strictly rigorous and complete; the interested reader is referred to (Bertsekas, 1987) and (Kumar, 1986). First, define the *optimal cost-to-go* function $V_{k+1}(C_{k+1})$ at time $k+1$ as:

$$V_{k+1}(C_{k+1}) = \min_{\mathrm{X}_{k+1},\ldots,\mathrm{X}_{N-1}} E\left\{-\alpha^{N-(k+1)}\gamma_N SF_N + \sum_{i=k+1}^{N-1} \alpha^{i-(k+1)} CC_k(i, X_i)\right\} \tag{13}$$

Function (13) represents the cost incurred by applying the portion of the optimal policy from period $k+1$ to period $N-1$, starting at period $k+1$ with capacity C_{k+1}. The explicit dependence of the function in equation (13) on the demand distribution was left out of the notation for simplicity.

Assuming the optimality of the cost-to-go function $V_{k+1}(C_{k+1})$, one can write the optimal cost-to-go function $V_k(C_k)$ at time *k*, for every *k*, as:

$$
\begin{aligned}
&V_k(C_k) = \min_{X_k} E\left\{CC_k(k, X_k) + \alpha \cdot E_{D_{k+1}}\left[V_{k+1}(C_{k+1})\right]\right\} \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad k = 0,1,\ldots,N-1 \\
&V_N(C_N) = -\gamma_N SF_N \qquad\qquad\qquad\qquad\quad k = N
\end{aligned}
\tag{14}
$$

Equations (14) are the so-called *optimality equations* for the optimal capacity-related reconfiguration problem. It can be proved - for an elegant proof the reader is referred to (Kumar, 1986) - that an optimal policy for this problem exists if and only if the minimum at (14) is achieved, for every k and for every C_k. In the following, an intuitive explanation of this last statement is provided.

Equations (14) can be rewritten by explicitly considering the optimal capacity expansion problem from the optimal capacity reduction problem, taking into account equations (10) and (11) – see (Asl and Ulsoy 2002a). Moreover, by substituting the definition of the control cost given in equation (1) into (14), and by denoting with $V_k^L(C_k)$ and $V_k^U(C_k)$ the optimal cost-to-go at time k of respectively the expansion and reduction problem, it yields, for every $k=0,\ldots, N$:

$$
\begin{cases}
V_k^L(C_k) = V_k^L(C_k) = \\
= \min\limits_{X_k \geq 0}\left\{E + e \cdot X_k + EOC_k(C_k) + \alpha E_{D_{k+1}}\left[V_{k+1}^L(C_{k+1})\right]\right\} \\
V_k^U(C_k) = V_k^U(C_k) = \\
= \min\limits_{X_k \leq 0}\left\{R + r \cdot X_k + EOC_k(C_k) + \alpha E_{D_{k+1}}\left[V_{k+1}^U(C_{k+1})\right]\right\}
\end{cases}
\tag{15}
$$

Defining $H_k^L(C_k) = V_k^L(C_k) + e \cdot C_k$ for the expansion problem and $H_k^U(C_k) = V_k^U(C_k) + r \cdot C_k$ for the reduction problem, and assuming, without loss of generality, that $E=R=0$, it yields, for every $k=0,\ldots, N$:

$$
\begin{aligned}
&H_k^L(C_k) = \\
&= \min_{X_k \geq 0} \left\{ \underbrace{EOC_k(C_k) + e \cdot (1-\alpha) \cdot C_{k+1} + \alpha \cdot E_{D_{k+1}} \left[H_{k+1}^L(C_{k+1}) \right]}_{F_k{}^L(C_k)} \right\} \\
&H_k^U(C_k) = \\
&= \min_{X_k \leq 0} \left\{ \underbrace{EOC_k(C_k) + r \cdot (1-\alpha) \cdot C_{k+1} + \alpha \cdot E_{D_{k+1}} \left[H_{k+1}^U(C_{k+1}) \right]}_{F_k{}^U(C_k)} \right\}
\end{aligned}
\qquad (16)
$$

It can be demonstrated that, for any compact set of the form [0, M_k], with $M_k < +\infty$, the minimization problems defined by equations (17), which are equivalent to the two separate system functionality expansion and reduction problems, are well-posed problems; this means that the minima in the right-hand sides of equations (17) are always attained, for every $k=0,1,\ldots,N-1$. The proof relies on the existence of a minimum for $EOC_k(C_k)$ over the same compact set (recall that all operating costs functions are continuous over [0, M_k], for any $M_k < +\infty$), on the convexity of $V_N^L(C_N)$ and of $V_N^U(C_N)$, as well as on the induction principle, and it is omitted here only for reasons of space. For the interested reader, (Asl and Ulsoy 2002a) provides a good reference to such a kind of proof.

To compute the value of the minimum of F_k^L, it is sufficient to first compute the local minimum of the same functions on each of the interested intervals, depending on whether definitions and equations for case *I* or case *II* (Figure 3) must be applied, and then to compute the global minimum as the minimum among the local ones. This local minimum always exists. The same holds for F_k^U. The following considerations on the profiles of F_k^L and F_k^U in each interval can help the reader in understanding how the solutions to the local optimization problems in each interval can be computed:

- Functions F_k^L and F_k^U of the forms D_1, D_3 and D_6 are all linear in C_k, for every k. This means that local minima in these intervals could only be one of the two extreme points of the same intervals.
- Functions F_k^L and F_k^U of the form D_6 are always strictly increasing monotonic in C_k, for every k (which is not always the case for F_k^L and F_k^U of the form D_1 or D_3). This means that the minimum in this kind of interval is always the lowest boundary of the interval, i.e. that both the optimal capacity expansion and reduction levels cannot rise up to $+\infty$. This is true in any case of practical interest.
- Functions F_k^L and F_k^U of the form D_2 are always convex in C_k, provided that $(\gamma_P\text{-}P\text{-}\gamma_S) \le 0$, which is a reasonable assumption, since $P \ge \gamma_P$ and $P, \gamma_P, \gamma_S \ge 0$ for any case of practical interest.
- Functions F_k^L and F_k^U of the form D_5 are always concave, provided that $(\gamma_m\text{-}\gamma_p) \ge 0$. This is also reasonable, given the meaning attributed to γ_m.
- Functions F_k^L and F_k^U of the form D_4 are generally both convex and concave, with an inflection point C_k* internal to the related interval.

Given the previous set of considerations, the optimal expansion and reduction levels L_k and U_k can be now defined symbolically, for every $k=0,\ldots,N\text{-}1$, in both cases I and II, as in the following:

$$L_k = \underset{C_k \in S_{k_{I,II}}}{\arg\min}\ F_k{}^L(C_k) \qquad U_k = \underset{C_k \in S_{k_{I,II}}}{\arg\min}\ F_k{}^U(C_k) \tag{17}$$

where:

$$S_{k_I} = [0, C_k{*}] \qquad S_{k_{II}} = [0, \Delta_k / \varepsilon_k] \tag{18}$$

L_k and U_k are indeed either one of the extreme points of the sub-intervals, or one of the local minima belonging to the same intervals. These minima can be computed, where they exist, as follows:

$$\left(\frac{\partial EOC_k}{\partial C_k}\right)_{D_i} + \kappa\cdot(1-\alpha)+\alpha\cdot\phi(k)\cdot\left(\kappa-\gamma_N\right)=0$$
$$\phi(k)=\begin{cases}1 & k=N-1\\ 0 & k\neq N-1\end{cases} \tag{19}$$

Where κ is equal to *e* or *r* for the expansion and reduction problems respectively. For instance, when L_k belongs to the interval where operating costs are defined by D_2, its value solves the following equation:

$$\Psi_k(L_k)=\frac{\left(\gamma_P-P-\gamma_S\right)\varepsilon_k+\left(\gamma_m-\gamma_P\right)\left(1-\varepsilon_k\right)+\gamma_H+e\left(1-\alpha\right)+\alpha\cdot\phi(k)\cdot\left(e-\gamma_N\right)}{\left(\gamma_P-P-\gamma_S\right)\varepsilon_k} \tag{20}$$

The procedure and the formulas above hold for any generic continuous univariate demand distribution Ψ_k. By letting $\tau_k\rightarrow 0$, i.e. $\varepsilon_k\rightarrow 1$, one obtains the same optimal values of the approach by (Asl and Ulsoy 2002a). Therefore, the optimal solutions by Asl and Ulsoy represent a particular case of the more complete approach presented in these pages, since the two approaches lead to the same solution when the ramp-up effect is negligible.

Figure 5 shows the F_k^L and F_k^U functions for the same case already shown in Figure 4: Example of profiles of the operating costs functions (*Case I*); the minima of these functions are respectively L_k and U_k..

All the formulas reported in equations (17) through (20) are valid, provided that $U_k \geq L_k$. Finally, the optimal policy for the capacity-related reconfiguration problem is presented in (21), by means of optimal boundaries based on the optimal system expansion and reduction levels L_k and U_k:

$$X_k^*(C_k)=\begin{cases}L_k-C_k & if \quad C_k<L_k\\ 0 & if \quad L_k\leq C_k\leq U_k\\ U_k-C_k & if \quad C_k>U_k\end{cases} \quad \text{for } k=1,\ldots,N-1 \tag{21}$$

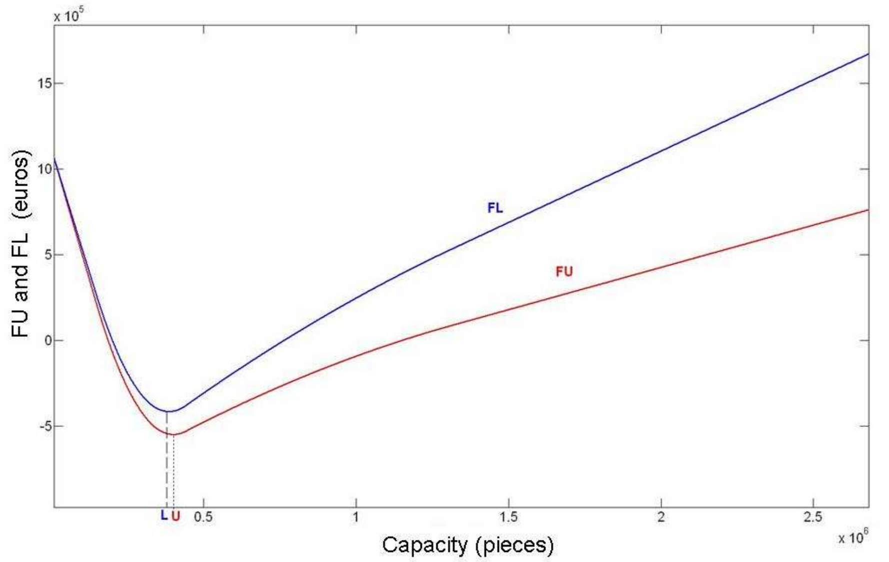

Figure 5: Profiles of the F^L_k and F^U_k for the case defined in Table 2

3.2 Interpretation of the optimal boundaries

The value of the difference between the optimal boundaries U_k and L_k - *optimal interval* in the following - can be interpreted as a measure of the likelihood of the need to reconfigure the production system. When this value is large, the system will not be often reconfigured, unless some particular scenario happens. On the opposite, the system may be frequently changed when this difference is small.

Figure 6 shows the iso-lines representing points with equal difference (U_k - L_k), as a function of the unit expansion and reduction costs. From the graph, one can see that the dimension of the optimal interval increases with the increase of *e* and with the decrease of *r*. Indeed, once given a fixed *r* (thus a fixed U_k), an increase in *e* implies a lower convenience in expanding the system functionality, i.e. in a lower optimal value for the expansion problem, namely L_k. The lower bound for such a decrease of L_k is represented by zero. Recall that the upper bound for L_k is represented by U_k. On the opposite, given a fixed *e* (thus a fixed L_k), an increase in *r* implies a higher convenience in reducing the system functionality, and thus in a lower U_k. The lower bound in this case is represented by L_k. The converse happens when the value of *r* is decreased.

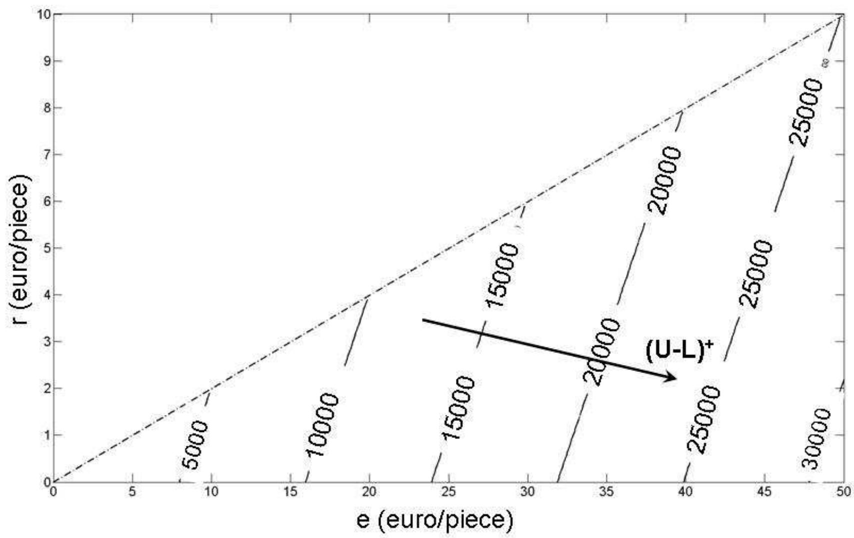

Figure 6: (U_k-L_k) iso-lines as a function of e and r

Figure 7 represents the optimal interval, again by means of the *(U_k - L_k,)* iso-lines, as a function of (Δ_k-δ_k), which can be taken as a first rough measure of demand variability in period *k*, and of τ_k, representing a timely measure of the weight of ramp-up in period *k*. It is possible to see that the dimension of the optimal interval grows both with the demand variability and ramp-up duration. This also follows from the equations of the model (sections 2.3 and 3.1). Further confirmations of the behaviour shown in Figure 7 are provided by the following considerations.

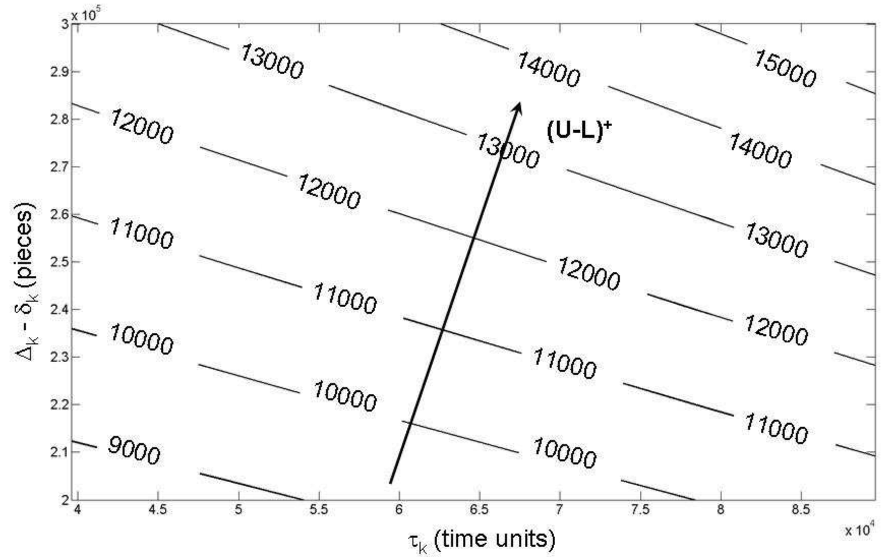

Figure 7: (U_k-L_k) iso-lines as a function of (Δ_k-δ_k) and τ_k

At first, given a value for τ_k (or ε_k), an increase in the variability of the product demand (Δ_k - δ_k) implies an increase in (U_k - L_k). To analytically prove that this is true in general, all the different closed-form values of U_k and L_k should be examined. For simplicity, a verification of this property in just a specific case is reported in the following. When the demand is uniformly distributed over [δ_k, Δ_k], formulas of case *I* apply, L_k = 0 and U_k belongs to [δ_k/ε_k , $\delta_k/(1-\varepsilon_k)$]. In such a situation, the operating cost functions are defined by formulas D_2 (Table 1) and U_k is defined by (22):

$$U_k = \frac{\delta_k}{\varepsilon_k} + \frac{(\gamma_P - P - \gamma_S)\varepsilon_k + (\gamma_m - \gamma_P)(1-\varepsilon_k) + \gamma_H + r(1-\alpha) + \alpha\cdot\phi(k)\cdot(r-\gamma_N)}{(\gamma_P - P - \gamma_S)\varepsilon_k{}^2}(\Delta_k - \delta_k) \quad (22)$$

Similar expressions could be reported for the other situations, i.e. for the rest of the D_i's, for both cases *I* and *II*. With a similar reasoning, one can see that, for a given (Δ_k - δ_k), an increase in τ_k implies an increase in (U_k - L_k).

Finally, one can easily notice that, based on the model parameters δ_k, Δ_k and ε_k at period *k*, the decision maker is already able to estimate the maximum dimension of the optimal interval, denoted here with $(U_k - L_k)^{MAX}$, without computing the values of L_k and U_k . In any case, it yields:

$$(U_k - L_k)^{MAX} \leq \frac{\Delta_k}{\varepsilon_k} \tag{23}$$

The meaning of $(U_k - L_k)^{MAX}$ can be described as follows. When $\tau_k \rightarrow 0$, i.e. when $\varepsilon_k \rightarrow 1$ (negligible ramp-up), $(U_k - L_k)^{MAX}$ cannot be larger than the maximum value forecasted for the product demand (namely Δ_k). In the worst case, it happens that $L_k = 0$ and $U_k = \Delta_k$.

Moreover, when $\tau_k \rightarrow T_k$, i.e. when $\varepsilon_k \rightarrow 1/2$ (non-negligible ramp-up), which happens when the ramp-up effect is important, $(U_k - L_k)^{MAX} \leq 2\Delta_k$. This upper bound is larger than the one for $\tau_k \rightarrow 0$, thus indicating that with an increase in the weight of the ramp-up effect, the optimal interval tends to widen, or equivalently that the decision maker considers a sort of safety margin in making the reconfiguration decision, compared to the situation in which the ramp-up effect is negligible.

4 Case Study

Here the aim is to assess the potential of the optimal capacity-related reconfiguration model presented in the previous pages. A set of measures is proposed to quantify:

- the advantage of using the model considering ramp-up (proposed here) with respect to the model not considering it (Asl and Ulsoy 2002a), when ramp-up occurs;
- the unexpected costs incurred due to the ramp-up phenomenon but not predictable without the proposed model;
- the overall impact of the ramp-up phenomenon on the system capacity-related reconfiguration problem.

A case study taken from the automotive sector is analyzed in the following. The case is provided by a firm mainly producing engine components and, in particular, it represents the situation of a flexible manufacturing line producing cylinder heads. The collected data are reported in Table 2. τ_k was considered equal to 6 months in the first period, and equal to 3 months in the following periods. This to take into account that the first ramp-up, taking place at the start-up of the system, is much more consistent than the ones eventually following.

Table 2. Data set for the presented real case (for confidential reasons the real data set was scaled by a common factor)

γ_S = 5.1 €/piece	γ_H = 0.2125 €/piece	γ_P = 17 €/piece	γ_N = 1.7 €/piece	γ_m = 18.7 €/piece
e = 20.6465 €/piece	r = 9.35 €/piece	β = 0.97	T = 1 year	P = 21.25 €/piece

The considered flexible line (Figure 8) has a linear layout, composed of four stations, performing respectively the machining operations (in a single phase), the deflashing of the workpieces, the washing of the machined pieces and a final leak test. The machining station is composed of identical CNC machining centers, and performs rough milling and basic drilling operations.

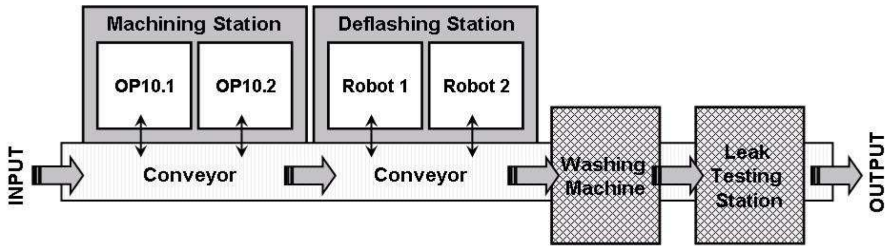

Figure 8: Layout of the considered flexible line

Figure 9 presents a multi-period analysis performed with reference to a 16-years planning horizon and realistic production volumes estimates. Forecasts of future expected demand values were available and uniform distributions were assumed at each period. The figure shows the profiles of system capacity when the optimal reconfiguration model considering production ramp-up (solid line) and the one not considering it (dotted line) are respectively applied. The minimum and maximum levels of product demand and the upper and lower limits provided by both models are also represented. Moreover, an initial capacity equal to the maximum demand forecasted for the first period is considered.

By analyzing the figure, one can easily notice that the optimal policy proposed in this paper leads in general to higher capacity levels. This is aligned with the previous discussion on the meaning of the optimal boundaries building up the optimal policy. Nevertheless, these capacity levels are optimal levels, while the ones provided by the model not considering ramp-up are not.

This last assertion is supported by the numbers in Table 3, where $V^{*}_{\pi^*}(C_o)$ and $V^{*}_{\pi(AU)}(C_o)$ represent the values of the multi-period cost function defined in Sub-section 2.4, resulting from the application of respectively the optimal policy proposed in the present paper and the one proposed by Asl and Ulsoy in their work. Finally, $V^{*,AU}(C_o)$ represents the value of the multi-period cost function, being at the basis of the work by Asl and Ulsoy, which assumes that the ramp-up does not exist. All values refer to the real case defined above, and C_o represents the initial system capacity.

Despite of the fact that all these values are positive, the profitability of the investment is not questionable here. Indeed, the firm operating the considered system usually asks the customers to assume a portion of the total investment. The actual contribution from the customer is the result of many contractual interactions, and it has a practical effect on the final setting of the product price. To remain as general as possible, in the presented case all costs are calculated by assuming that the customer's contribution in the total investment cost is equal to zero.

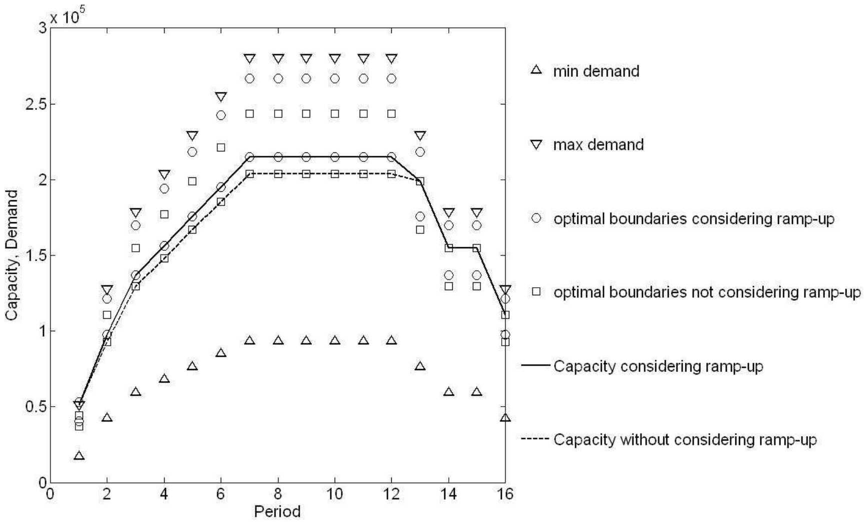

Figure 9: Multiple period case

Despite of the fact that all these values are positive, the profitability of the investment is not questionable here. Indeed, the firm operating the considered system usually asks the customers to assume a portion of the total investment. The actual contribution from the customer is the result of many contractual interactions, and it has a practical effect on the final setting of the product price. To remain as general as possible, in the presented case all costs are calculated by assuming that the customer's contribution in the total investment cost is equal to zero.

A first comparison between $V^*_{\pi^*}(C_o)$ and $V^*_{\pi(AU)}(C_o)$, numerically quantifies the practical advantage derived by the use of the optimal reconfiguration model considering ramp-up, when ramp-up occurs. Results state that by using the proposed model, the overall multi-period cost incurred during the considered time horizon is 5.6% less than the one incurred by applying the optimal policy presented by Asl and Ulsoy.

Moreover, the difference between $V^*_{\pi(AU)}(C_o)$ and $V^{*,AU}(C_o)$ can be used as a first measure of the unexpected costs incurred because of the ramp-up, when the optimal policy not considering ramp-up is applied. In the present real case, the repeated production ramp-ups incurred in the first periods lead to an overall multi-period cost which is 9.9% higher than expected: this means that not considering ramp-up can seriously jeopardize the investment.

Table 3. Comparison of different policies and models

	$V^{*}_{\pi*}(C_o)$	$V^{*}_{\pi(AU)}(C_o)$	$V^{*,AU}(C_o)$
	0.788 M€	0.832 M€	0.757 M€
Cost function considers ramp-up	YES	YES	NO
Optimal policy considers ramp-up	YES	NO	NO

Finally, the difference between $V^{*}_{\pi*}(C_o)$ and $V^{*,AU}(C_o)$ can be used as a first measure of the impact of the ramp-up phenomenon on the reconfiguration problem, being it the difference between the optimal values of the overall multi-period function provided by the application of the two different models to the same production problem. In the case of the flexible manufacturing line, this quantity states that this impact is, in terms of percentage costs, 4.1% more than the optimal value of the - by far ideal - case where ramp-up does not happen.

5 Conclusions and future developments

This paper presented a solution, based on Markov decision Theory, for the system capacity-related reconfiguration problem under stochastic market demand. The solution was proposed as optimal boundaries representing the optimal capacity expansion and reduction levels, explicitly considering production ramp-up. First, an insight into the meaning of the presented optimal policy was provided. Then, it was shown that by ignoring the ramp-up effect, the derived policies both are non-optimal and could also lead to significant underestimation of the overall reconfiguration and management costs. Future developments will lead in particular to the development of a more complete model for optimal capacity-related reconfiguration, taking into consideration product evolution scenarios from both technological and production perspectives.

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